

# H-mode pedestal turbulence in DIII-D and NSTX using BOUT++ code\*

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# Abstract

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In this work, we will report BOUT++ simulations for H-mode pedestal instabilities and turbulent transport. For DIII-D H-mode discharges, the BOUT++ peeling-ballooning ELM model including electron inertia was used to analyze the ideal linear stability and ELM dynamics. The beta scan is carried out from a series of self-consistent MHD equilibria generated from EFIT by varying pressure and/or current. For typical tokamak pedestal plasmas with high temperature and low collisionality, we found that the collisionless ballooning modes driven by electron inertia are unstable in the H-mode pedestal and have a lower beta threshold than ideal peeling-ballooning modes, which are the triggers for Edge Localized Modes. Thus, collisionless (electron inertia) ballooning modes might be responsible for H-mode turbulence transport when the pedestal is stable to peeling-ballooning modes. BOUT++ calculations also show that NSTX Elm stability boundaries are sensitive to flow shear profile. Attempts are underway to calculate nonlinear turbulence and transport in H-mode discharges due to the non-ideal effects.

# The Nonlinear System of Equations for Simulating Non-Ideal MHD Peeling-Ballooning Modes

$$\frac{\partial \tilde{\omega}}{\partial t} + \mathbf{v}_E \cdot \nabla \tilde{\omega} = B_0 \nabla_{\parallel} \tilde{J}_{\parallel} + 2\mathbf{b}_0 \times \boldsymbol{\kappa}_0 \cdot \nabla \tilde{p} + \mu_{i,\parallel} \partial_{\parallel 0}^2 \tilde{\omega} + \mu_{i,\perp} \nabla_{\perp}^2 \tilde{\omega}, \quad (1)$$

Here  $\nabla_{\parallel} F = B \partial_{\parallel} (F/B)$  for any  $F$ ,  $\partial_{\parallel} = \partial_{\parallel 0} + \tilde{\mathbf{b}} \cdot \nabla$ ,  $\tilde{\mathbf{b}} = \tilde{\mathbf{B}}/B = \nabla \tilde{A}_{\parallel} \times \mathbf{b}_0/B$ ,  $\partial_{\parallel 0} = \mathbf{b}_0 \cdot \nabla$ ,  $\boldsymbol{\kappa}_0 = \mathbf{b}_0 \cdot \nabla \mathbf{b}_0$ , ion pressure  $P_i$  and total pressure  $P = P_i + P_e$ .

$$\frac{\partial P}{\partial t} + \mathbf{v}_E \cdot \nabla P = \chi_{\parallel} \partial_{\parallel 0}^2 P, \quad (2)$$

$$\frac{\partial \tilde{A}_{\parallel}}{\partial t} = -\nabla_{\parallel} \Phi + \frac{\eta}{\mu_0} \nabla_{\perp}^2 \tilde{A}_{\parallel} - \frac{\eta_H}{\mu_0} \nabla_{\perp}^4 \tilde{A}_{\parallel}, \quad (3)$$

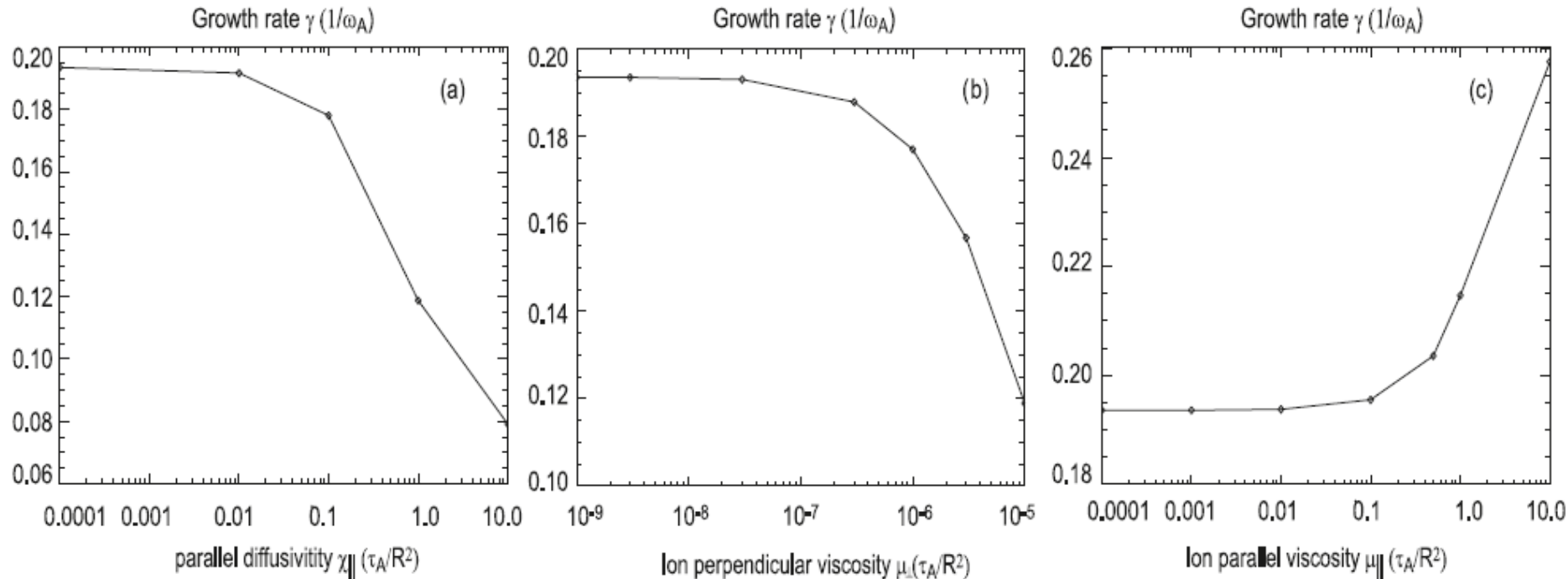
$$\tilde{\omega} = \frac{n_0 M_i}{B_0} \left( \nabla_{\perp}^2 \tilde{\phi} + \frac{1}{n_0 Z_i e} \nabla_{\perp}^2 \tilde{p}_i \right), \quad \Phi = \tilde{\phi} + \Phi_0, \quad (4)$$

$$P = \tilde{p} + P_0,$$

$$J_{\parallel} = J_{\parallel 0} - \frac{1}{\mu_0} \nabla_{\perp}^2 \tilde{A}_{\parallel}, \quad \mathbf{v}_E = \frac{1}{B_0} (\mathbf{b}_0 \times \nabla_{\perp} \Phi). \quad (5)$$

This simple set of reduced two-fluid equations effectively bypasses the issue of the gyroviscous cancellations in simulations while the important diamagnetic effect is retained in the second term of the generalized vorticity expression.

# The effect of transport coefficients on linear P-B instabilities.

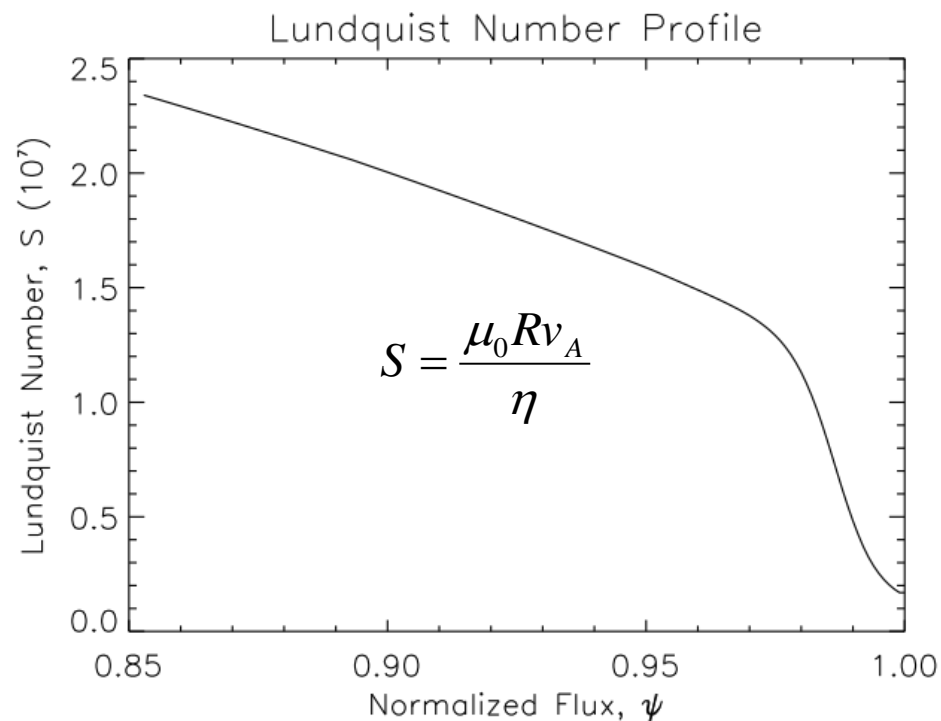
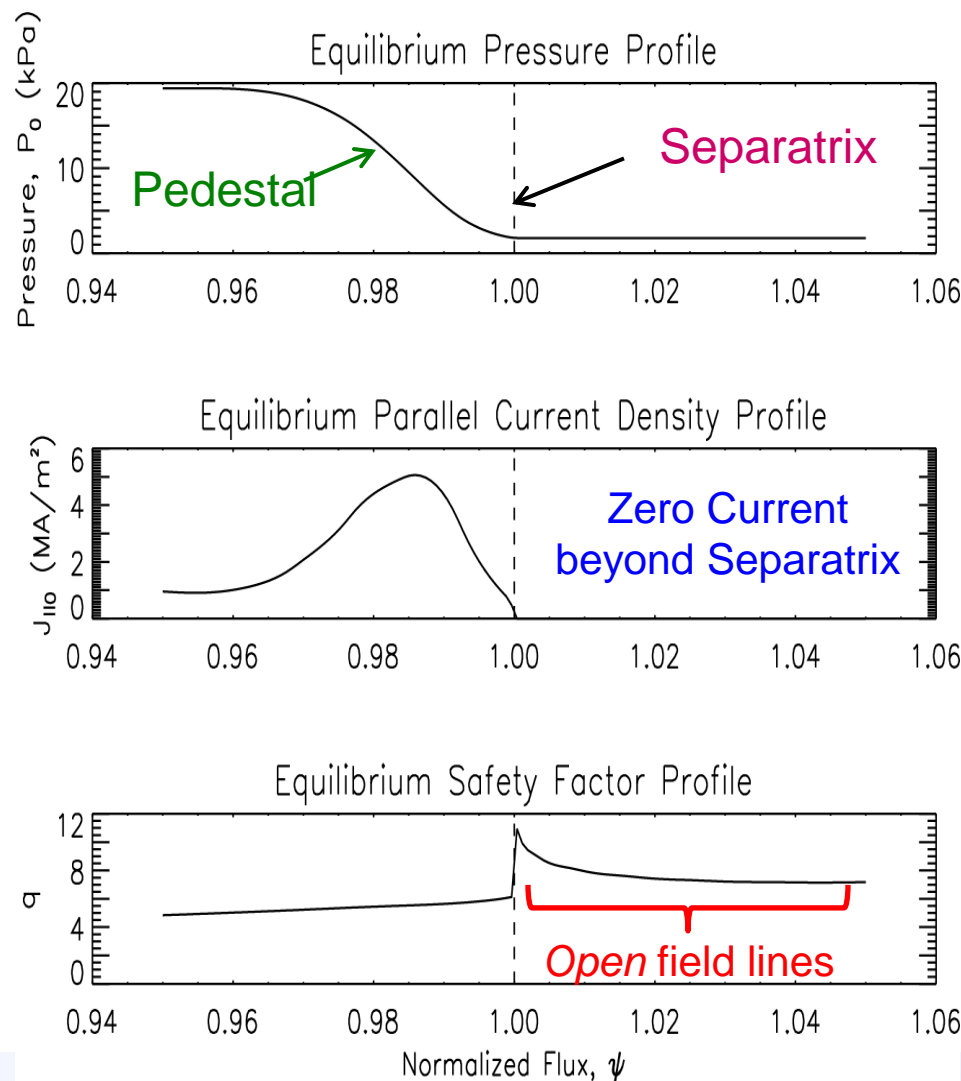


The growth rate of the  $n = 15$  eigenmode versus various transport coefficients with the  $E \times B$  drift and diamagnetic drift for  $S = 10^8$  and  $\alpha_H = 10^{-4}$ : (a) the parallel diffusivity  $\chi_{||}$ , (b) the ion perpendicular viscosity  $\mu_{i,\perp}$  and (c) the ion parallel viscosity  $\mu_{i,\parallel}$ .

For ITER pedestal parameters  $T_{e,ped} \simeq 4.5$  keV,  $n_{e,ped} \simeq 5 \times 10^{19} \text{ m}^{-3}$ ,  $\chi_{e,\parallel}^{SH} \simeq 2.62 \times 10^{11} \text{ m}^2 \text{ s}^{-1}$  and  $\chi_{e,\parallel}^{SH}/D_A \simeq 794$ , while  $\chi_{e,\parallel}^{FL} \simeq v_{Te} q_{95} R \simeq 1.16 D_A$ . Similarly, for typical pedestal plasma parameters,  $\mu_{i,\perp} \simeq (0.1 - 1) \text{ m}^2 \text{ s}^{-1}$  as radial thermal diffusivity with the assumption that the turbulent Prandtl numbers are close to unity. Namely,  $\mu_{e,\perp}/\chi_{e,\perp} \sim \mu_{i,\perp}/\chi_{i,\perp}$  and  $\chi_{e,\perp} \simeq \chi_{i,\perp}$ , which yields  $\mu_{i,\perp}/D_A \simeq (0.3-3) \times 10^{-8}$ , the impact of the perpendicular ion viscosity on the growth rate is negligibly small.

Equations (1)–(5) are solved using a field-aligned (flux) coordinate system  $(x, y, z)$  with shifted radial derivatives. Differencing methods used are fourth-order central differencing and third-order WENO advection scheme. The resulting difference equations are solved with a fully implicit Newton–Krylov solver: Sundials CVODE package. Radial boundary conditions used are  $\tilde{\omega} = 0$ ,  $\nabla_{\perp}^2 \tilde{A}_{||} = 0$ ,  $\partial \tilde{p}/\partial \psi = 0$  and  $\partial \tilde{\phi}/\partial \psi = 0$  on the inner radial boundary;  $\tilde{\omega} = 0$ ,  $\nabla_{\perp}^2 \tilde{A}_{||} = 0$ ,  $\tilde{p} = 0$  and  $\tilde{\phi} = 0$  on outer radial boundary. The domain is periodic in the parallel coordinates  $y$  (with a twist-shift condition) and in  $z$  (toroidal angle).

# C-Mod Equilibrium EDA H-Mode Parameters used as BOUT++ Input (1110201023.00900)

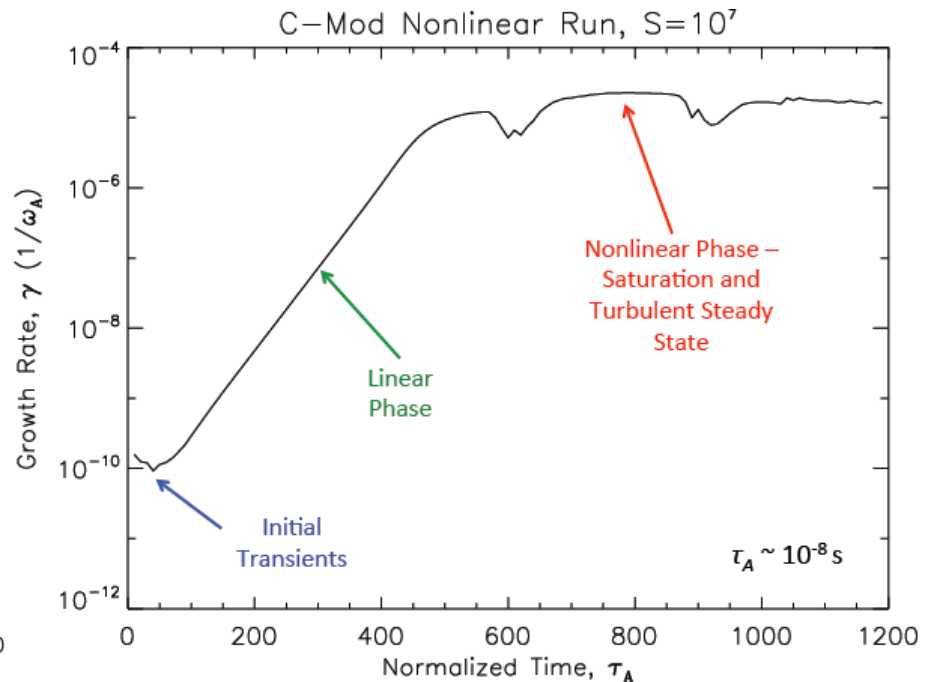
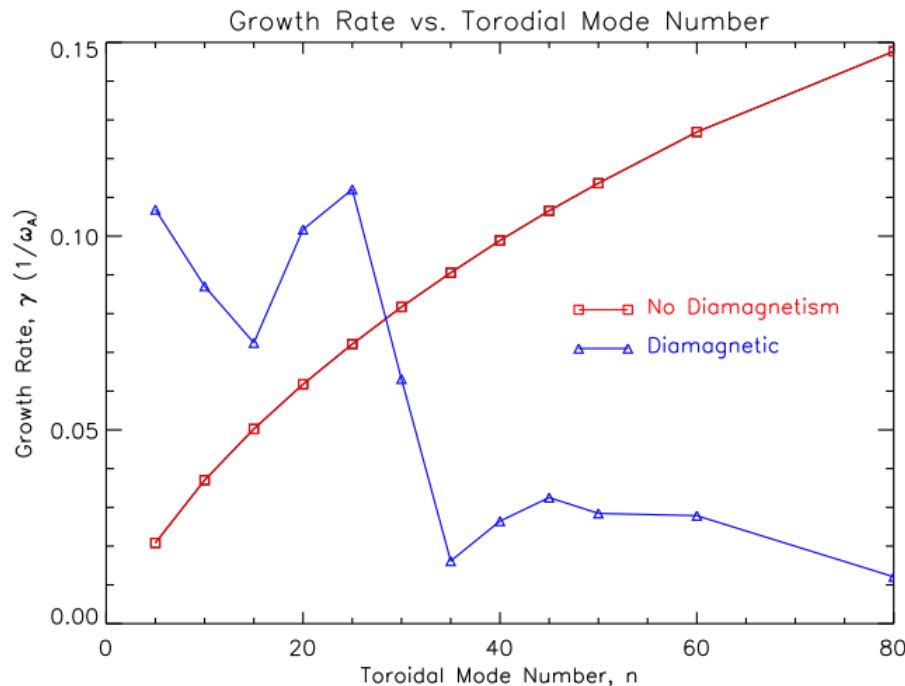


Lundquist Number ( $S$ ) is a dimensionless ratio of the resistive diffusion time to the Alfvén time

–  $S \sim 10^7$  in C-Mod EDA pedestal

# BOUT++ Calculations Show C-Mod EDA H-Modes Resistively Unstable

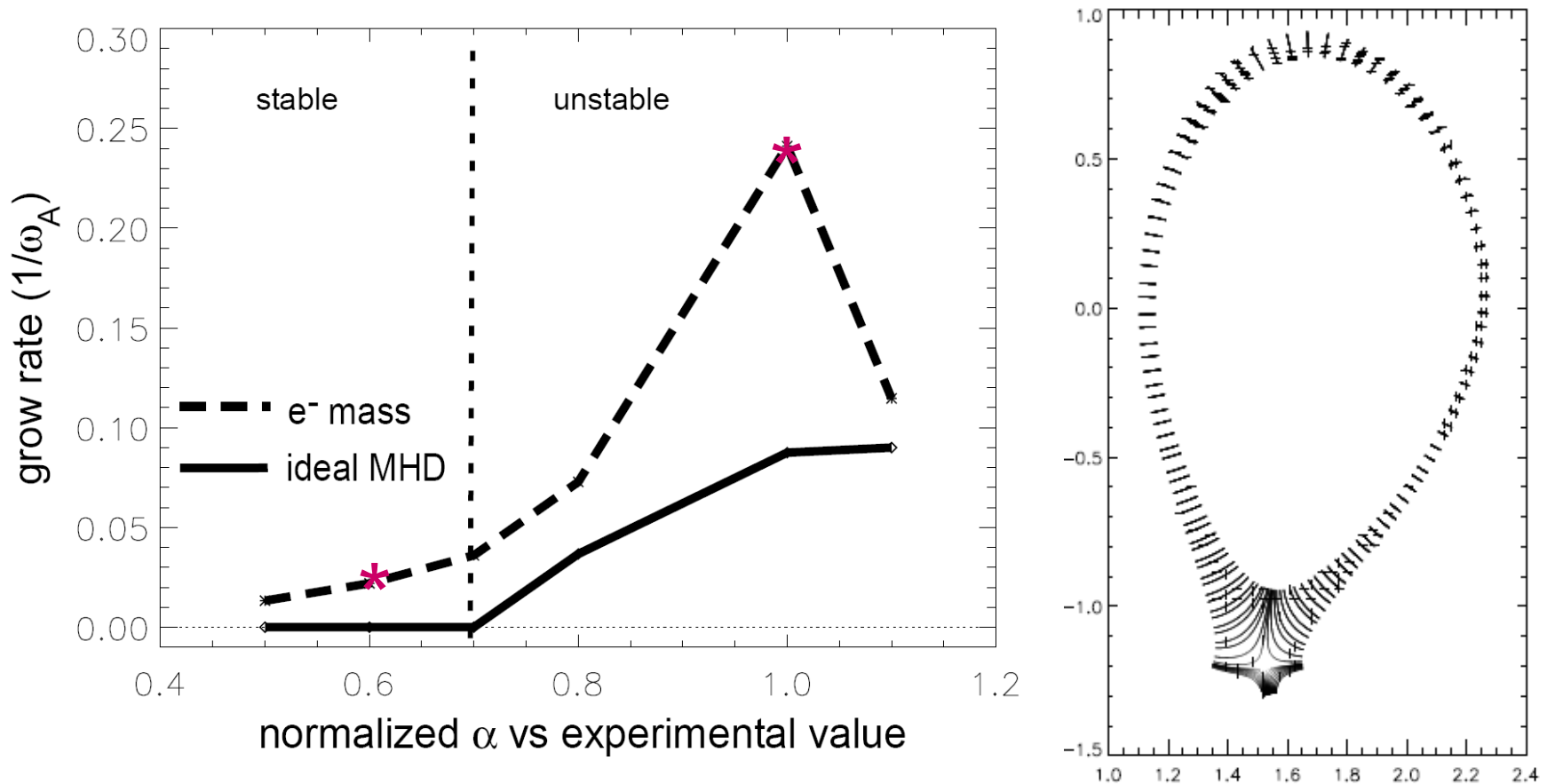
Alcator  
C-Mod



**BOUT++ calculations show that Diamagnetic Effects Damp Higher Mode Numbers, yielding the growth rate peaks at  $n=25$ , consistent with measurements.**

**Preliminary Nonlinear Simulations have begun --- Mode Saturation and Turbulent Steady-State have been Observed. Comparisons with experimental measurements will begin.**

# BOUT++ simulations for DIII-D ELMy H-mode shot #131997 at reduced $J_{||}$

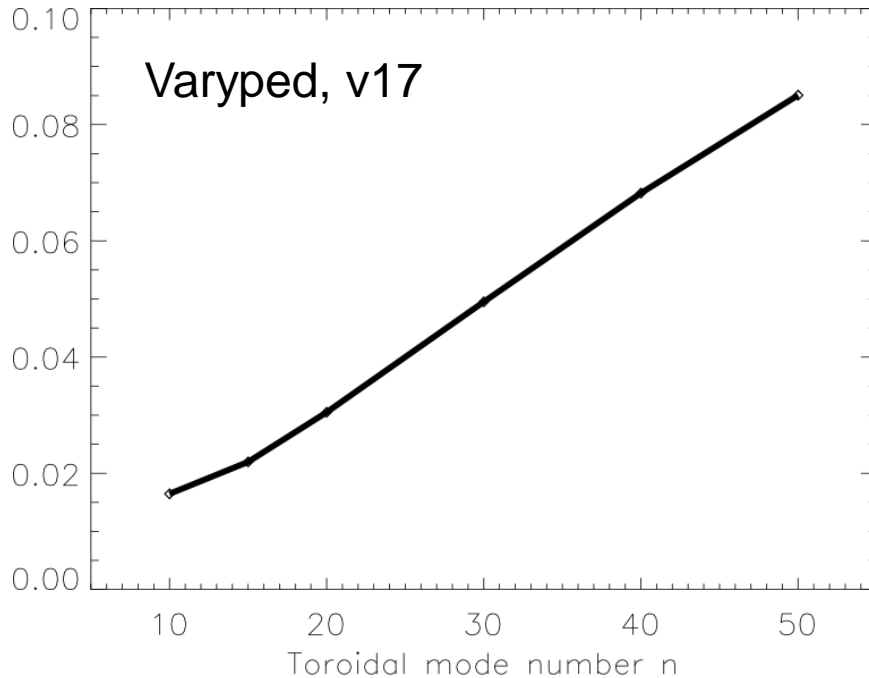


- ✓ Ideal MHD stability boundary is consistent with infinite-n BALLOO code
- ✓ Inclusion of  $e^-$  inertial eliminates the stability boundary

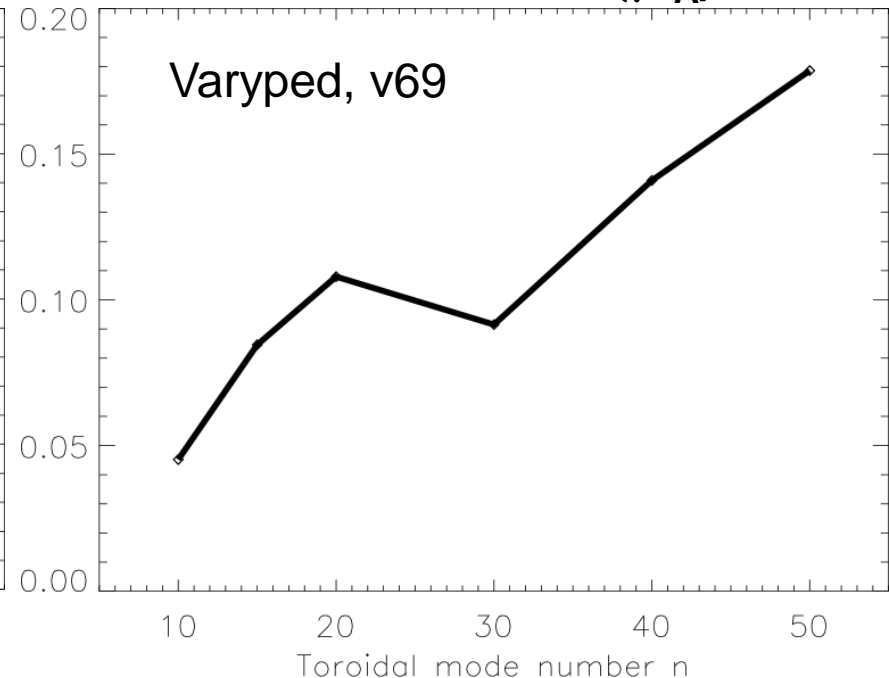
# BOUT++ simulations for DIII-D ELMy H-mode shot #131997 at reduced $J_{\parallel}$

Varyped:  $P_{0,v17}=0.6P_{0,exp}$ ,  $P_{0,v69}=P_{0,exp}$

Growth rate ( $\gamma\tau_A$ )



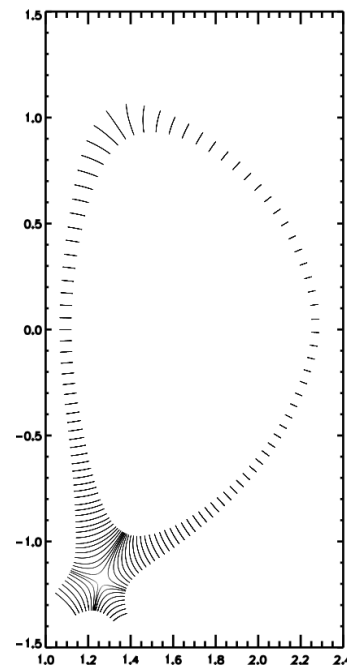
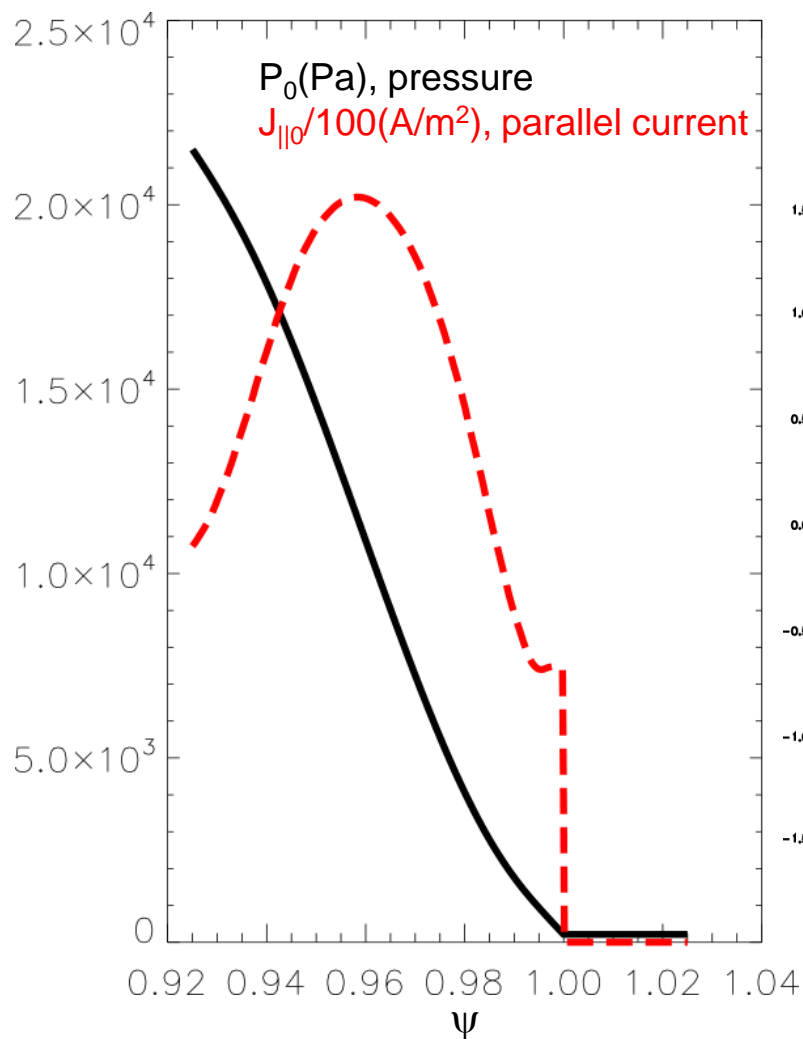
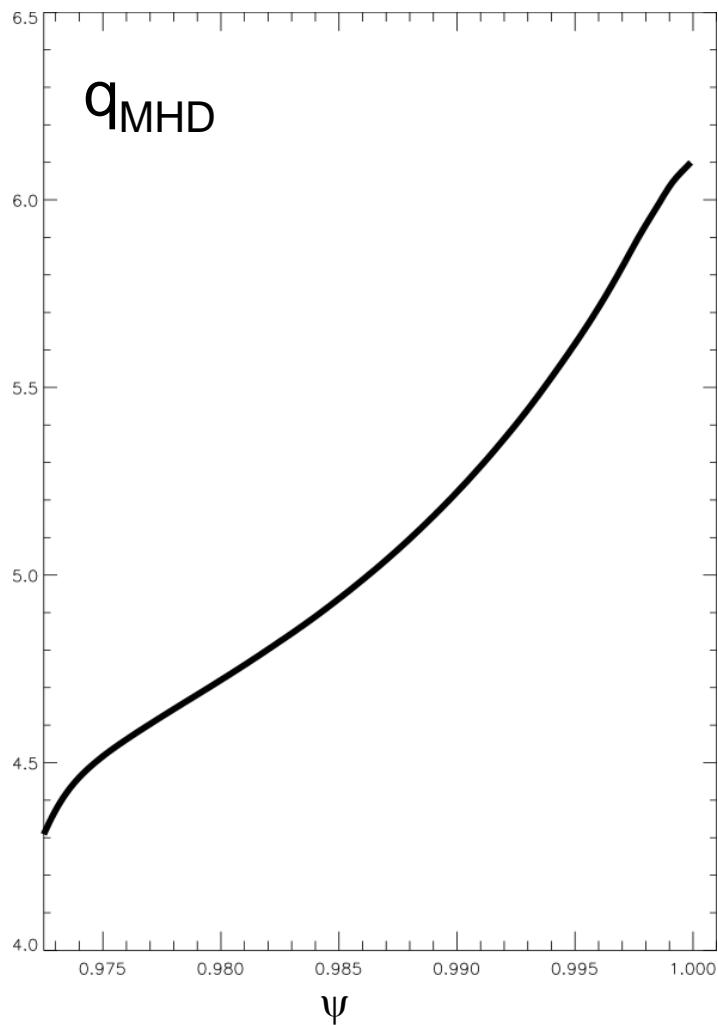
Growth rate ( $\gamma\tau_A$ )



✓ Inclusion of  $e^-$  inertial eliminates or reduces the ion diamagnetic stabilization

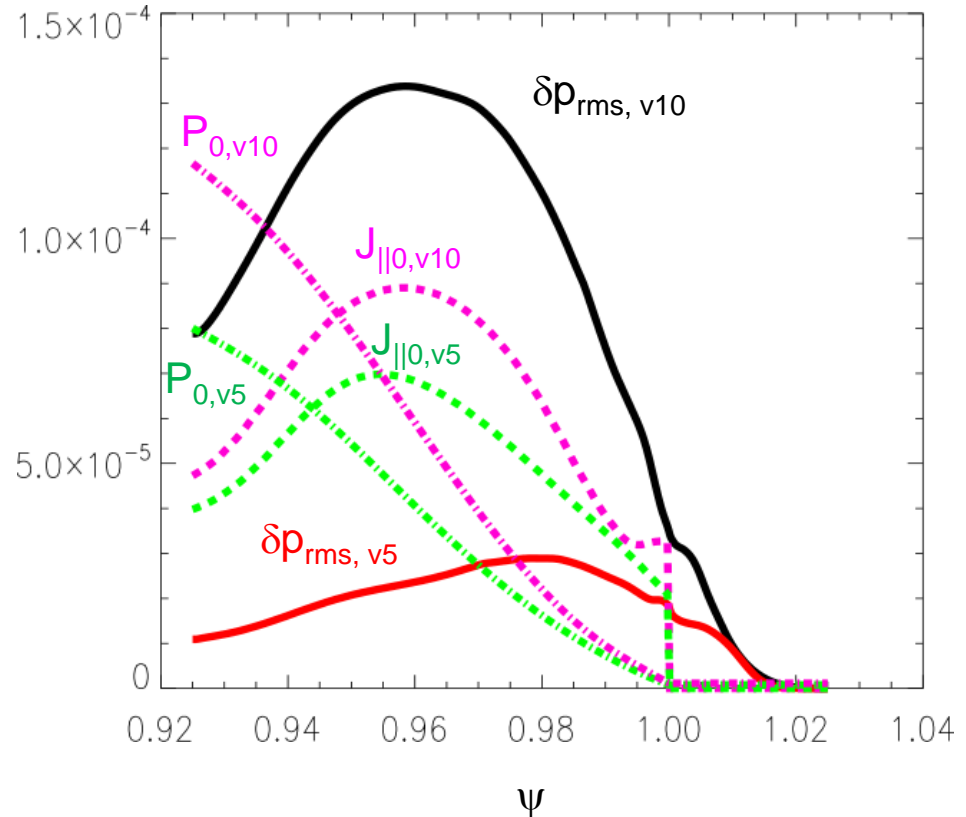
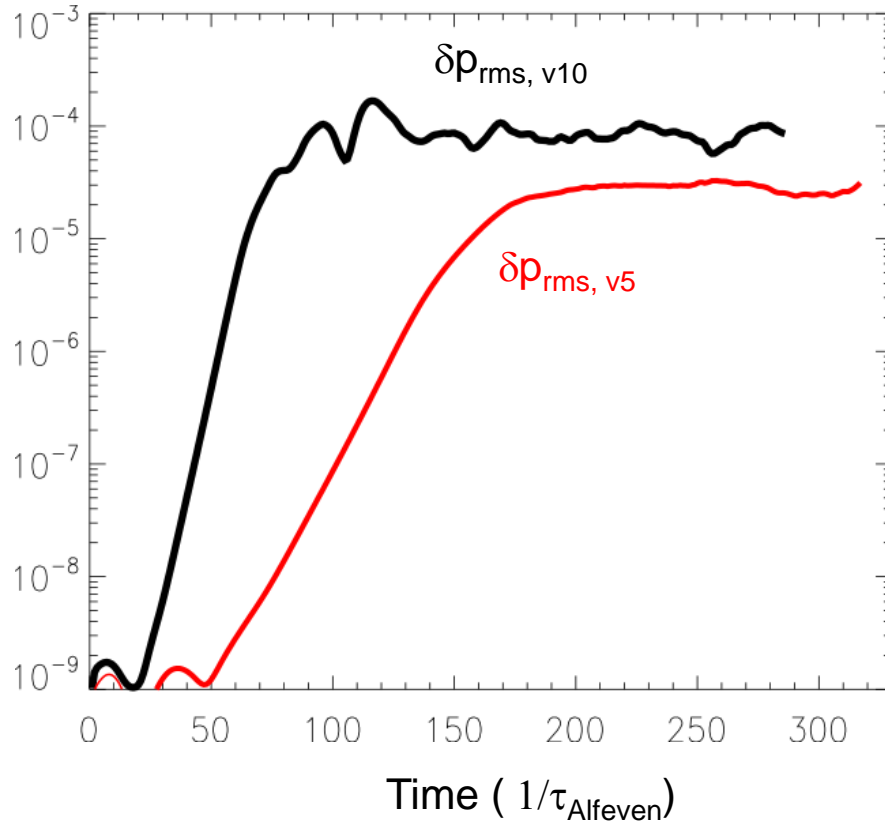


# BOUT++ nonlinear simulations for DIII-D H-mode shot #132016 at $t=3034\text{ms}$ & $I_p=1.49\text{MA}$



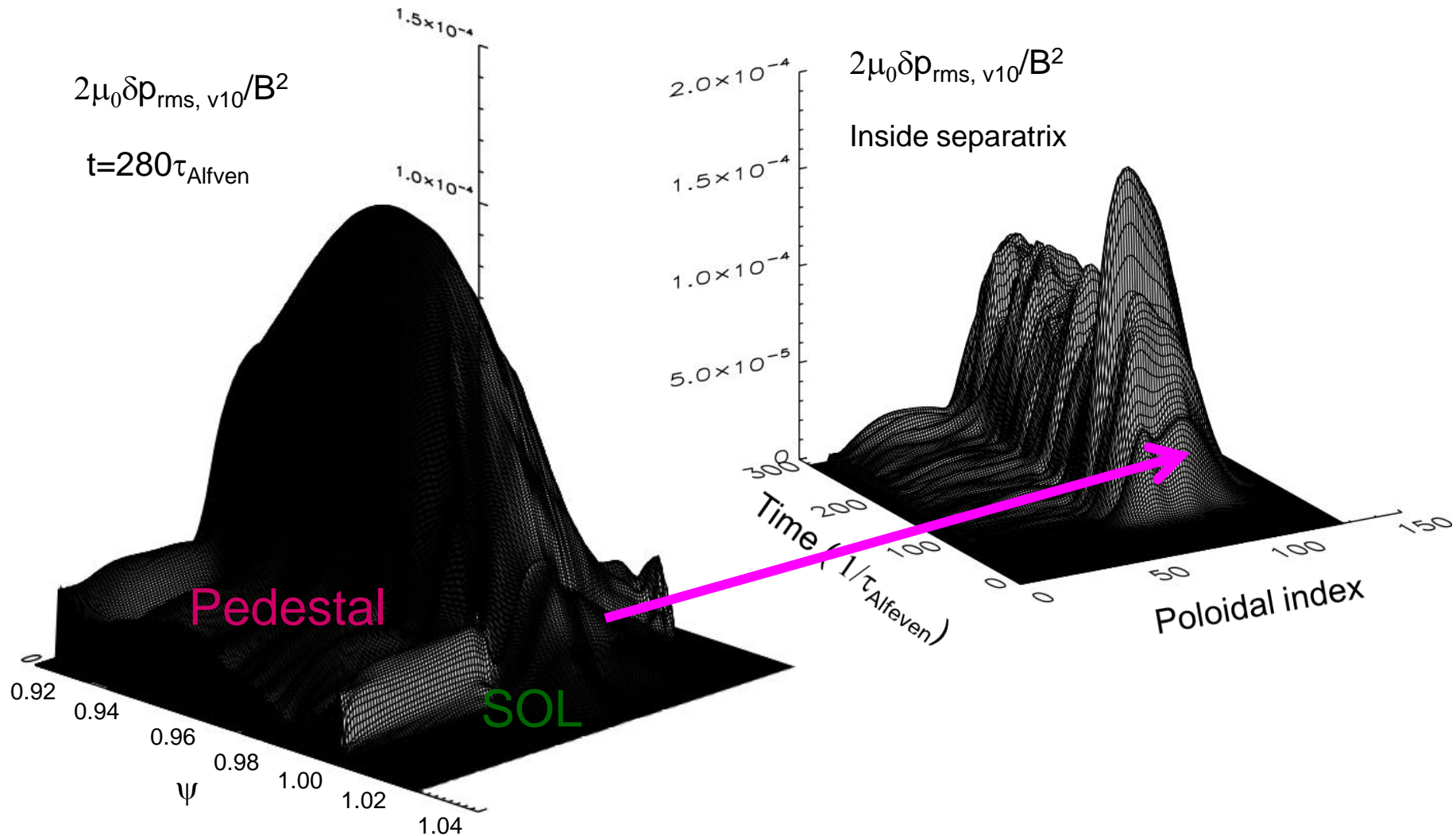
# BOUT++ nonlinear simulations for DIII-D H-mode shot #132016 at $t=3034\text{ms}$ & $I_p=1.49\text{MA}$

Varyped:  $P_{0,v5}=P_{0,\text{exp}}$ ,  $P_{0,v10}=1.5P_{0,\text{exp}}$



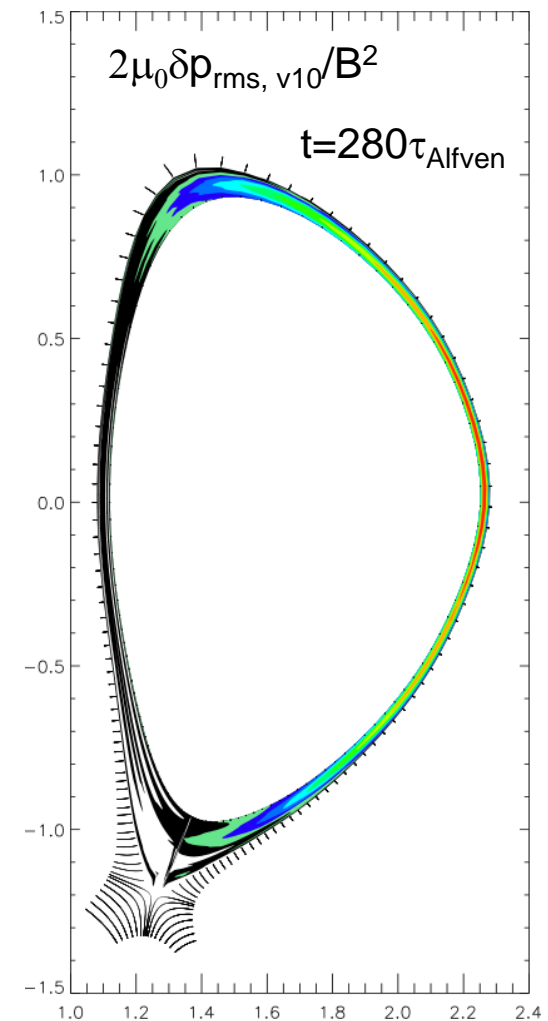
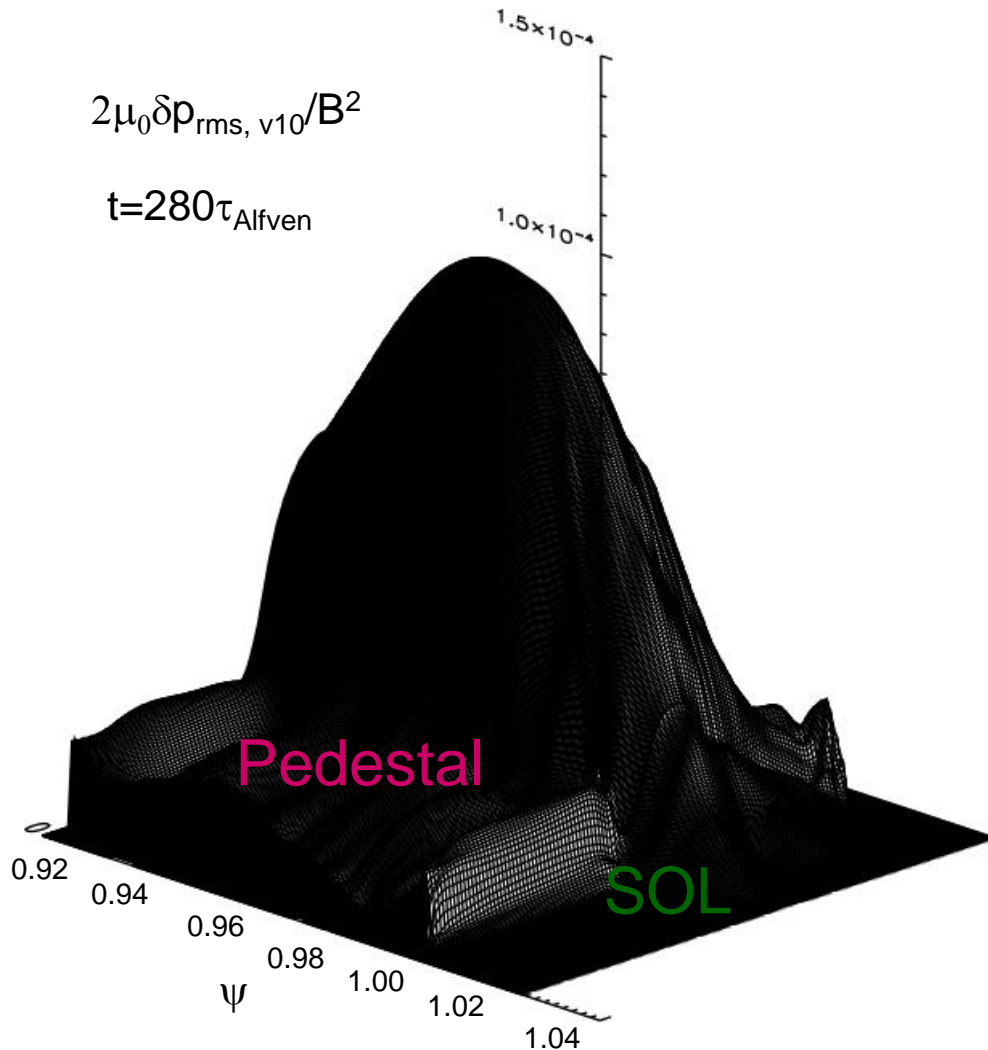
✓ Inclusion of  $e^-$  inertial eliminates the stability boundary

# BOUT++ nonlinear simulations for DIII-D H-mode shot #132016 at $t=3034\text{ms}$ & $I_p=1.49\text{MA}$



✓ Keeping ballooning mode eigen-function from linear to nonlinear phase

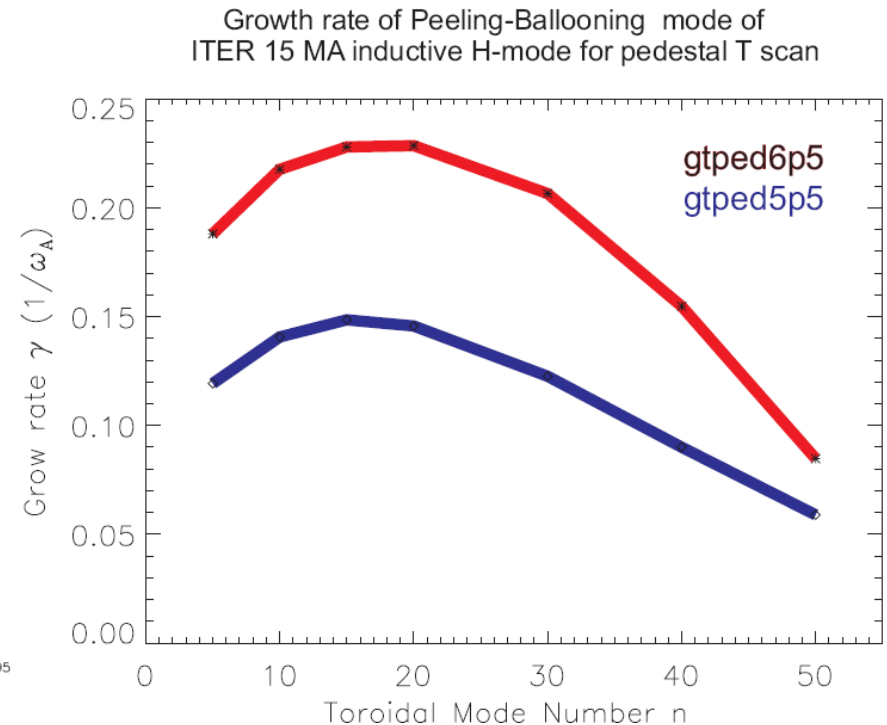
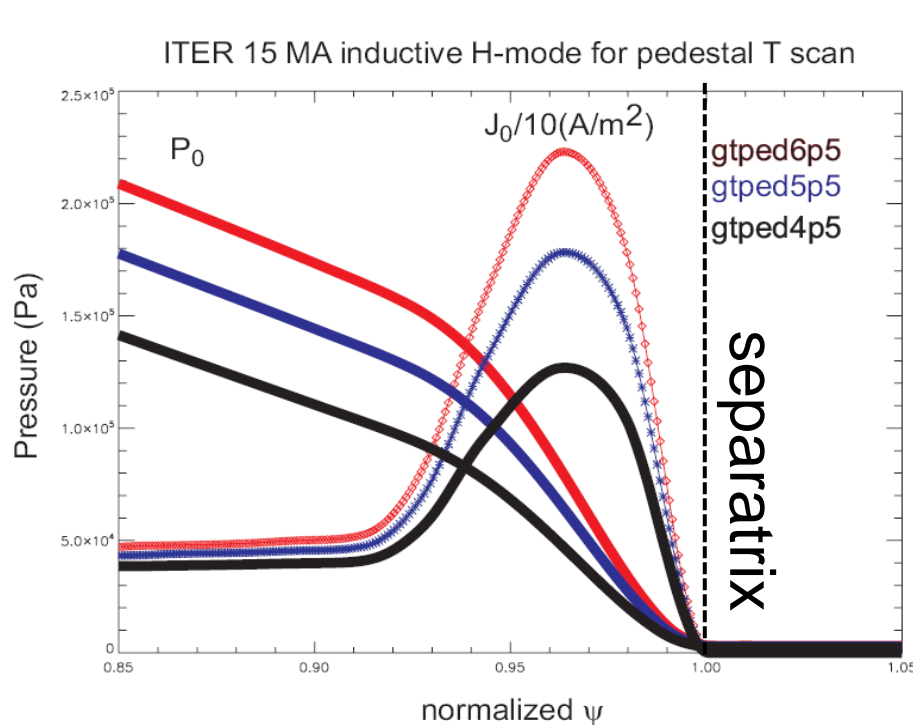
# BOUT++ nonlinear simulations for DIII-D H-mode shot #132016 at $t=3034\text{ms}$ & $I_p=1.49\text{MA}$



✓ X-point magnetic shear limits the poloidal extent of perturbation on low field side.

# BOUT++ simulations for one of the latest designs of the ITER 15 MA inductive ELMy H-mode scenario (under the burning condition)

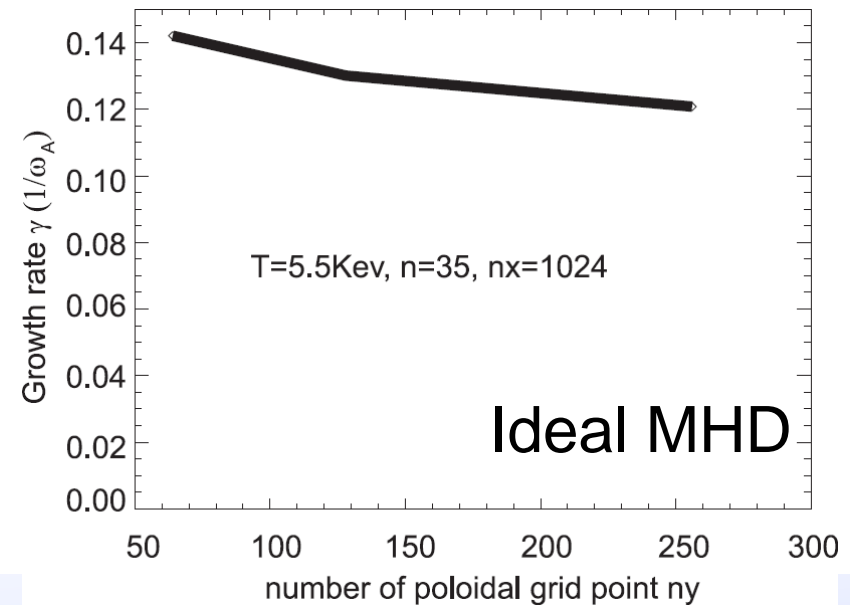
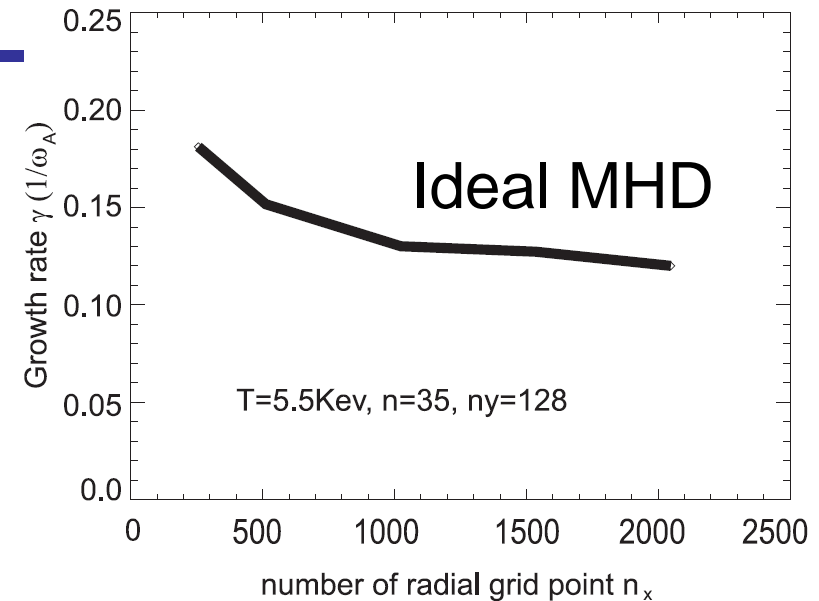
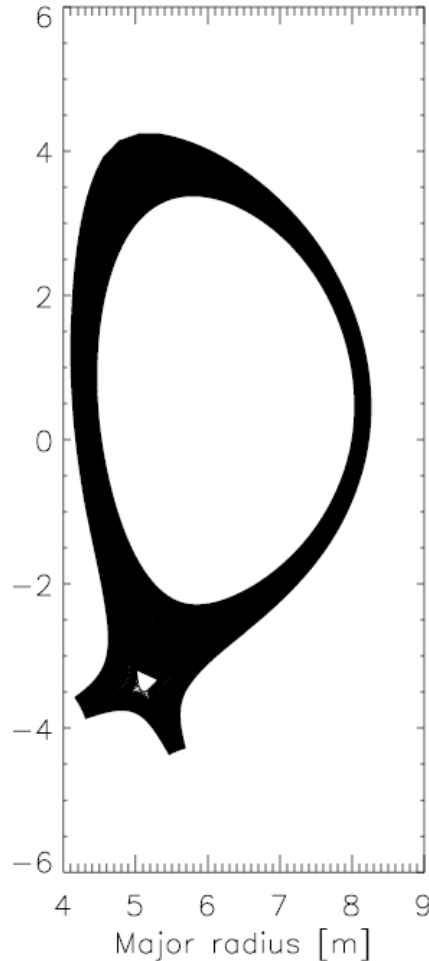
- Simulations starting from equilibrium generated by the CORSICA code.



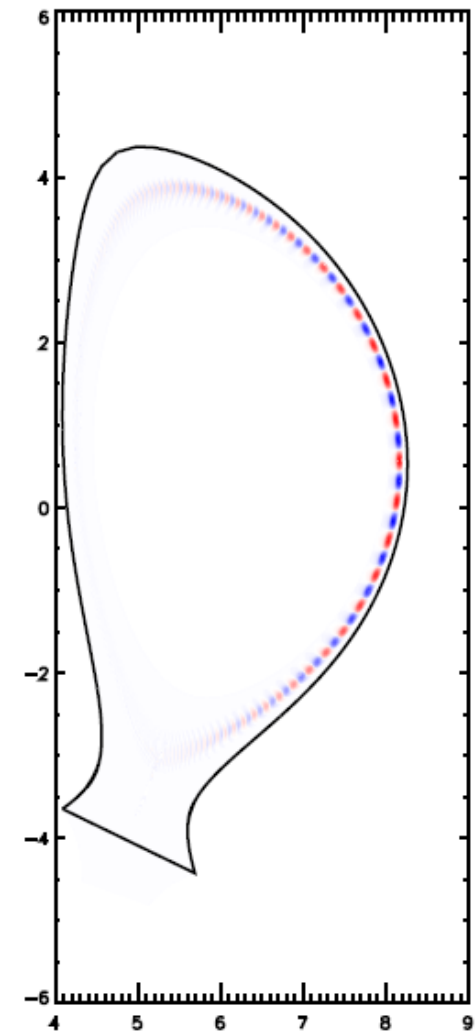
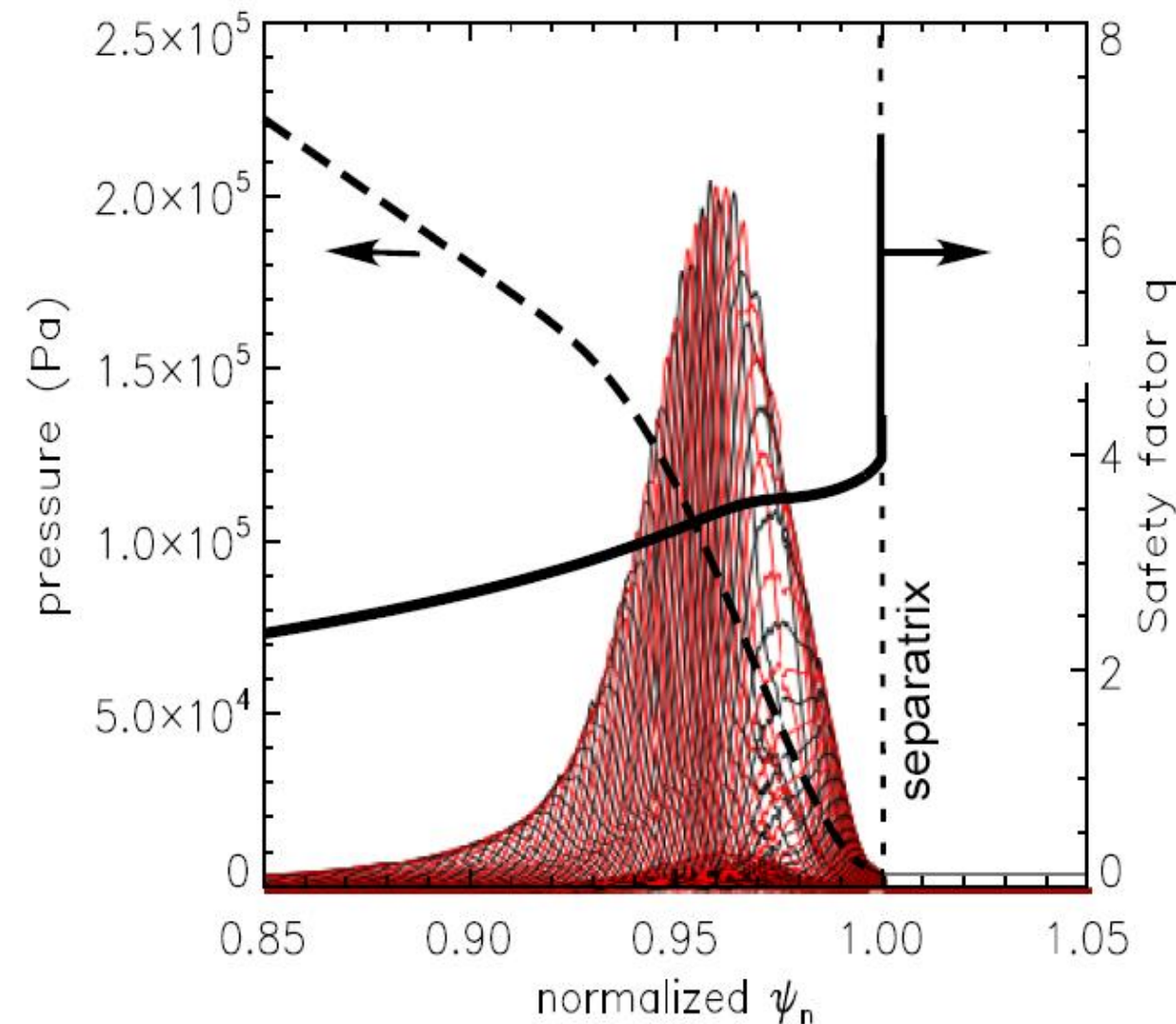
- Marginal unstable pedestal case,  $T_{\text{ped}}=5.5\text{keV}$ ,  $n_{\text{max}}=15$
- The calculations impact previous ITER ELMy H-mode scenario design as it was based on the pedestal height  $T_{\text{ped}}=4.5\text{keV}$

# BOUT++ simulations for one of the latest designs of the ITER 15 MA inductive ELMy H-mode scenario

It is numerical challenge to simulation ITER divertor geometry, requiring high resolutions  $n_x > 1000$ ,  $n_y > 100$ , even for linear mode.

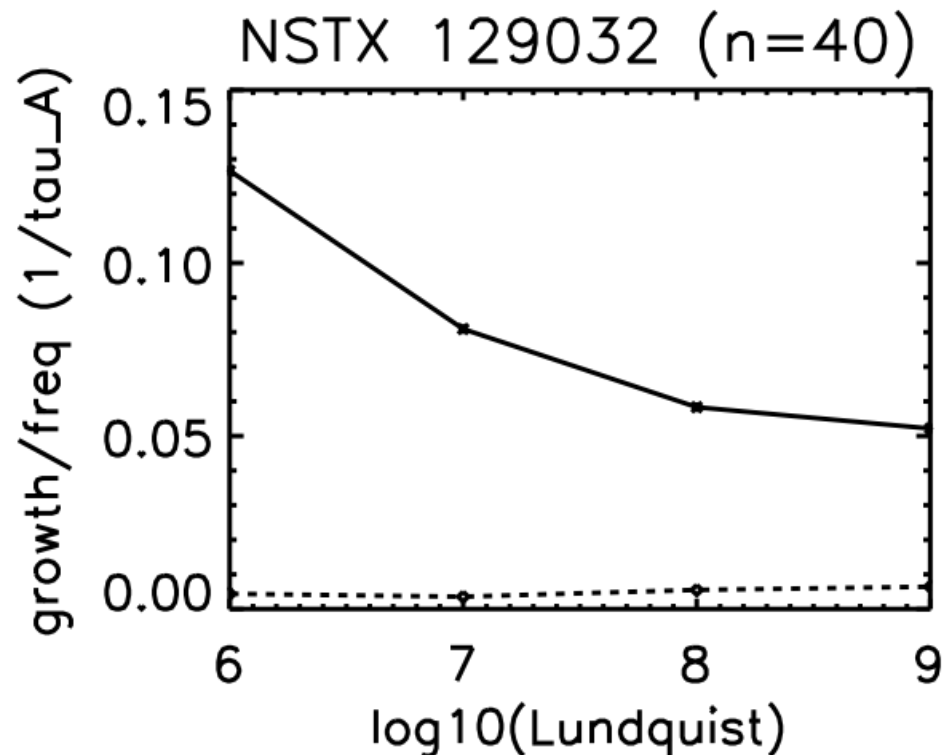


# BOUT++ simulations show radial and poloidal mode structures and for the ITER 15 MA inductive ELMy H-mode scenario

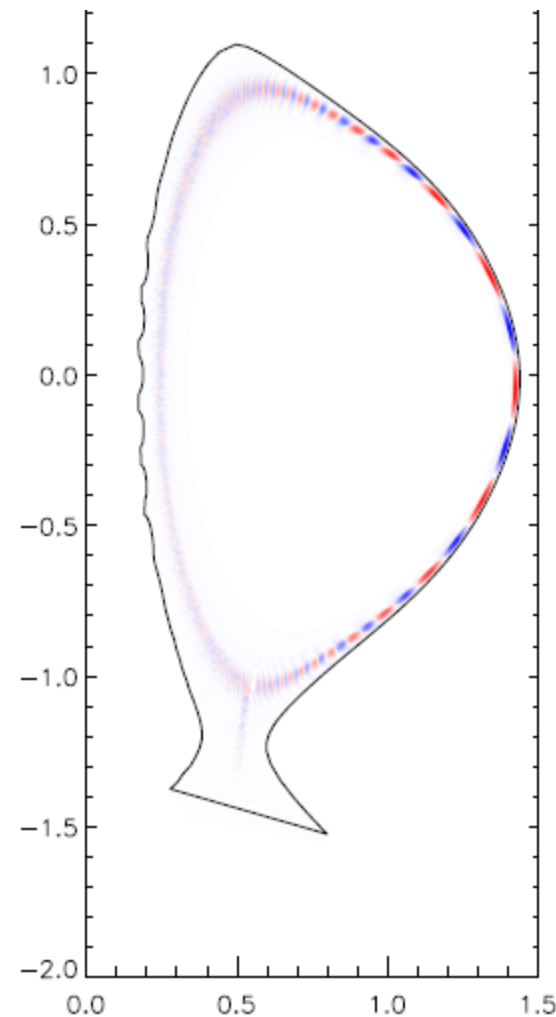




# BOUT++ Calculations Show NSTX discharge 129032 Resistively Unstable



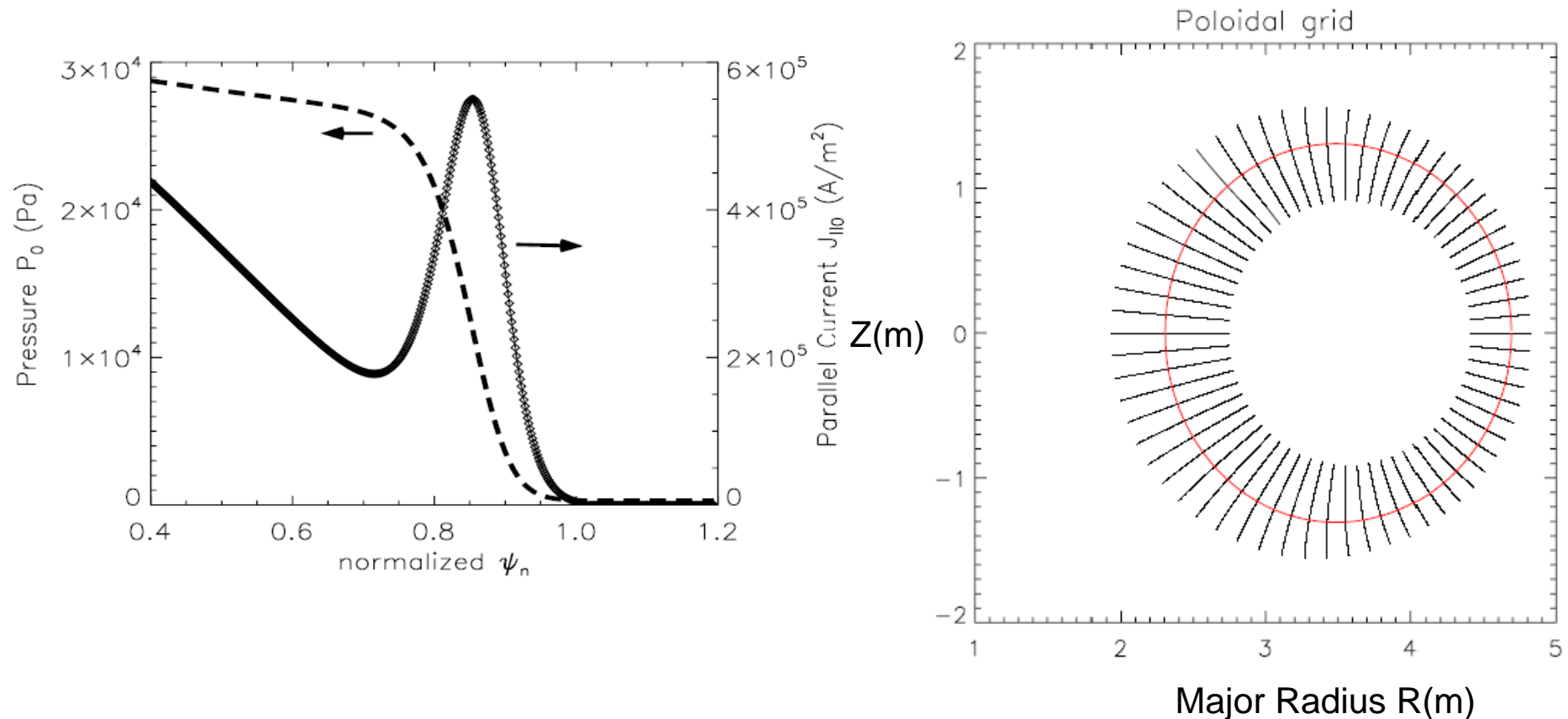
- ✓ With assumption that  $V_{\text{ExB}} = V_{\text{diam}}$ , all modes are stabilized.
- ✓ The detailed flow profile does matter for this discharge





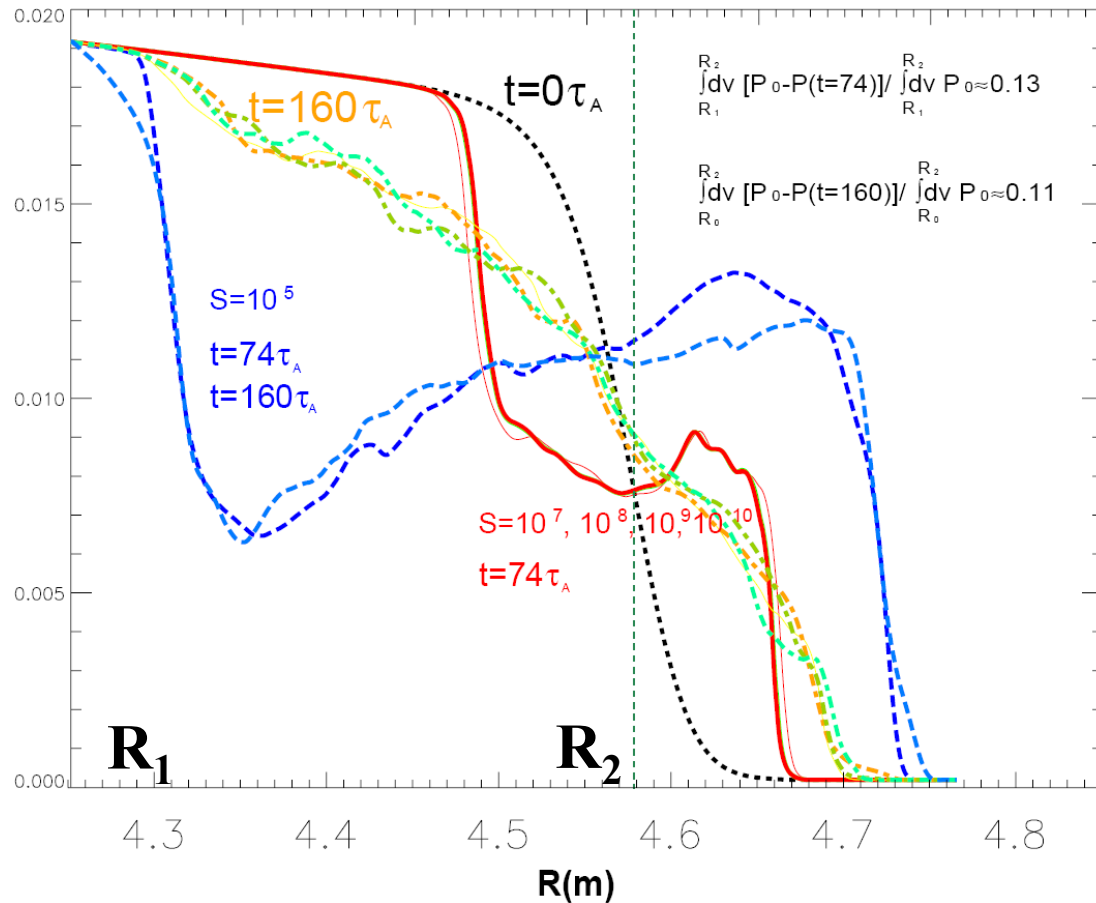
# Magnetic Reconnection and Pedestal Collapse during ELMs

Equilibrium current and pressure profiles used as BOUT++ input



# Flux-surface-averaged pressure profile $2m_0 \langle P \rangle / B^2$ vs $S$ with $S_H = 10^{12}$

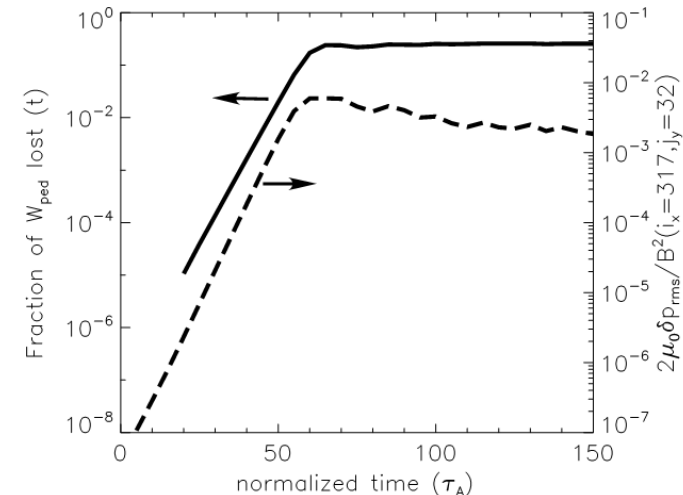
low  $S \rightarrow$  large ELM size, ELM size is insensitive when  $S > 10^7$



$$\text{ELM size} = \Delta W_{\text{ped}} / W_{\text{ped}}$$

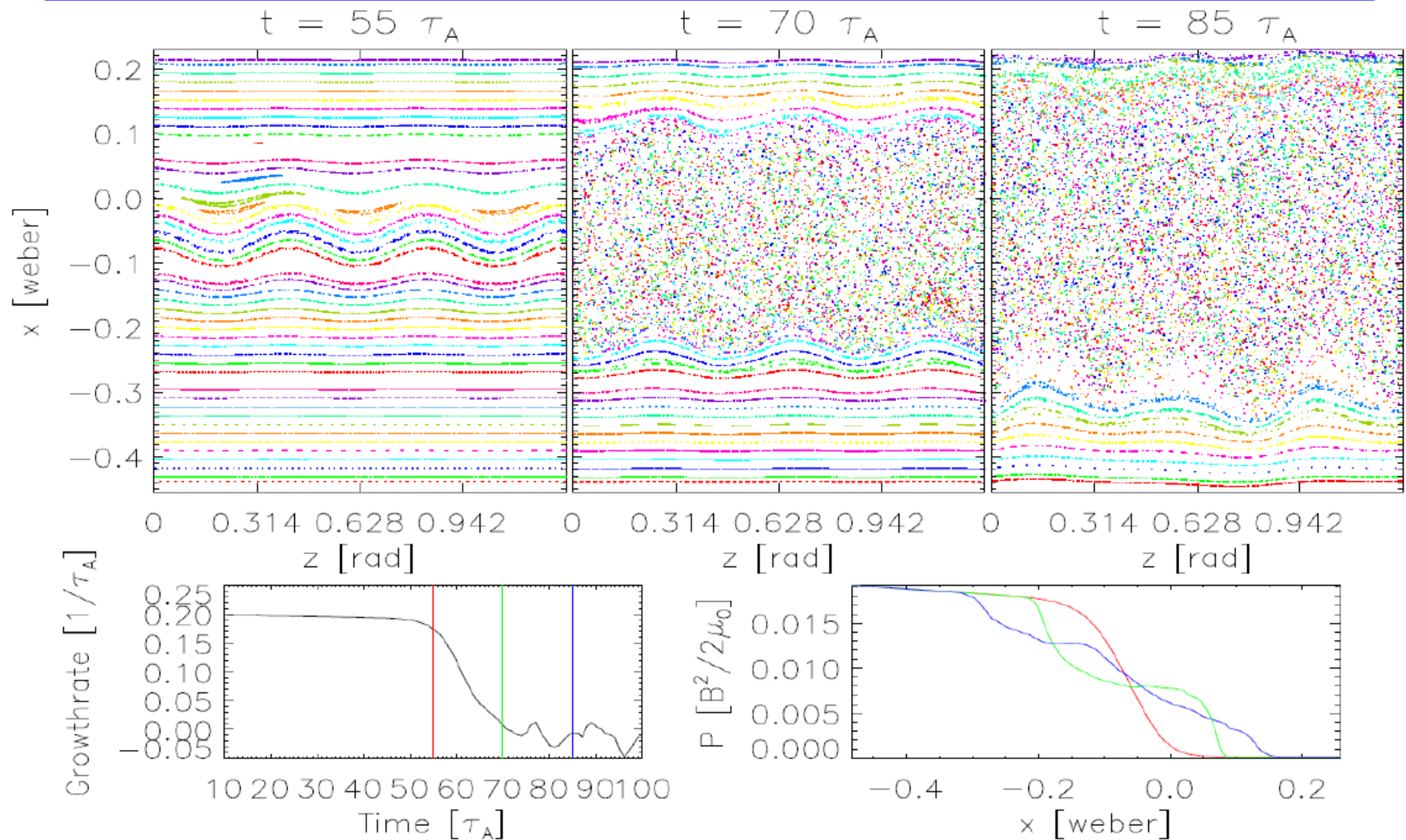
$\Delta W_{\text{ped}}$  = the ELM energy loss

$W_{\text{ped}}$  = pedestal stored energy



- (1) a sudden collapse: **P-B modes  $\rightarrow$  magnetic reconnection  $\rightarrow$  bursting process**
- (2) a slow backfill as a turbulence transport process

For  $S=10^8$ ,  $S_H=10^{12}$ , the reconnection region is small and the collapse is limited.



# Role of the hyper-resistivity on nonlinear ELM simulations.

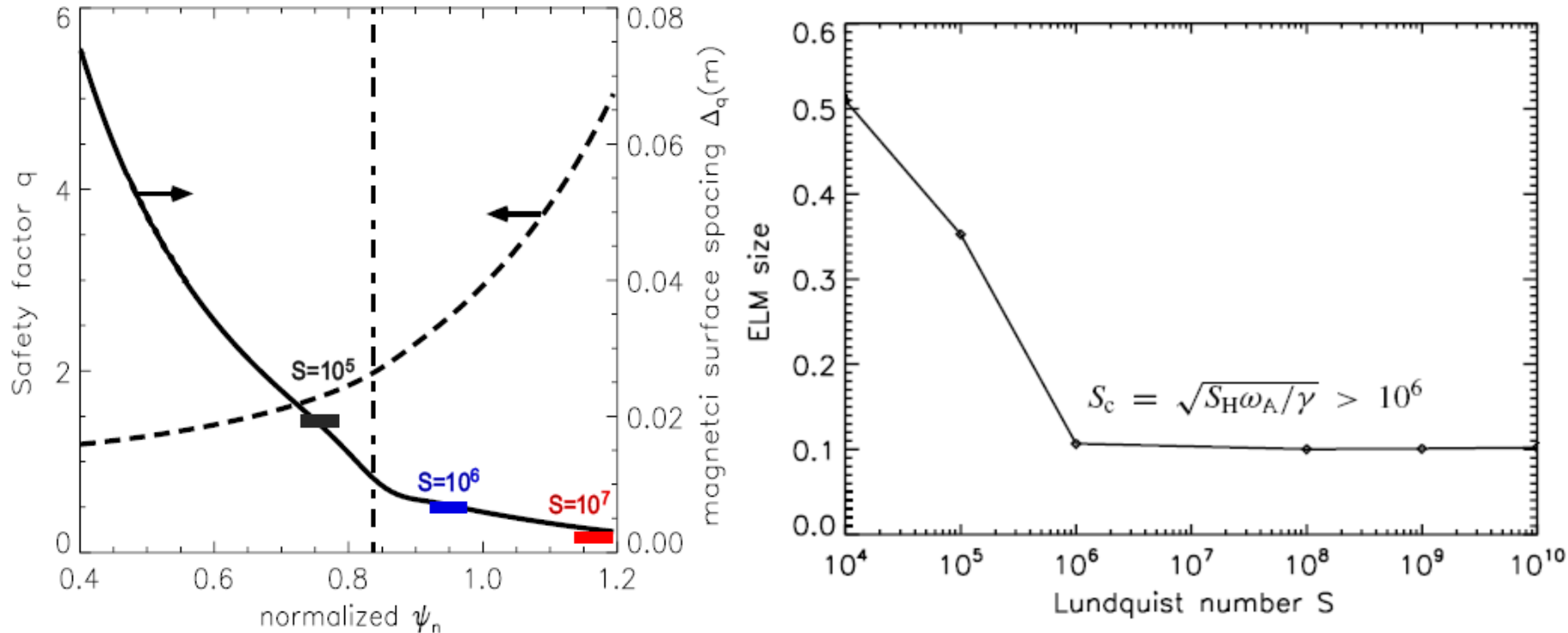


Figure 6. ELM sizes versus Lundquist number  $S$  with  $S_H = 10^{12}$ .

$$\frac{\partial \hat{A}_{\parallel}}{\partial \hat{t}} = \hat{\nabla}_{\parallel} \hat{\Phi} + \frac{1}{S} \hat{\nabla}_{\perp}^2 \hat{A}_{\parallel} + \frac{1}{S_H} \hat{\nabla}_{\perp}^4 \hat{A}_{\parallel}$$

ideal MHD term contains  $\nabla_{\parallel 0} = k_{\parallel} q R_0 = m - nq$

# Equilibrium flow shear model

$$\frac{\partial \varpi}{\partial t} + (\underbrace{\mathbf{V}_{EP0} + \mathbf{V}_{EV0}}_{\text{Total convection flow}}) \cdot \nabla \varpi + \underbrace{\mathbf{V}_1 \cdot \nabla \varpi_0}_{\text{Kelvin-Helmholtz term}} = B_0^2 \nabla_{\parallel} J_{\parallel} + 2\mathbf{b}_0 \times \kappa \cdot \nabla P_1$$

Diamagnetic convection flow
Net flow

$$\varpi = \frac{\rho_0}{B_0} \left( \nabla_{\perp}^2 \phi + \frac{1}{n_0 Z_i e} \nabla_{\perp}^2 P \right)$$

Diamagnetic drift

## ●Diamagnetic effects:

➤ **Diamagnetic convection flow:**  $\mathbf{E} \times \mathbf{B}$  flow that balances diamagnetic flow, is determined by pressure profile, introduces negative electric field  $\Phi_{dia0}$ ;

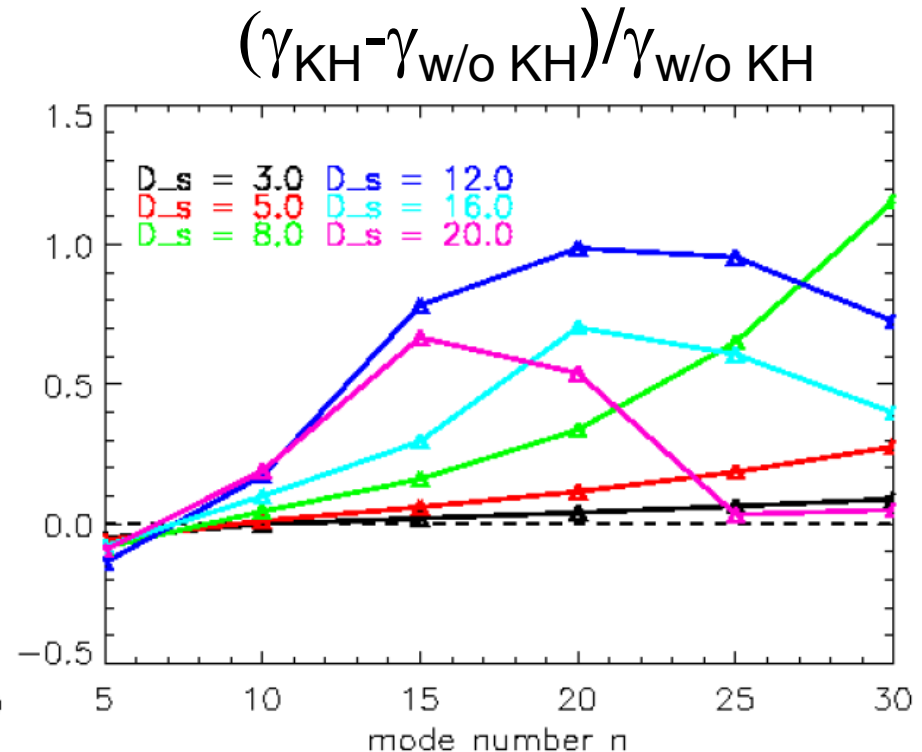
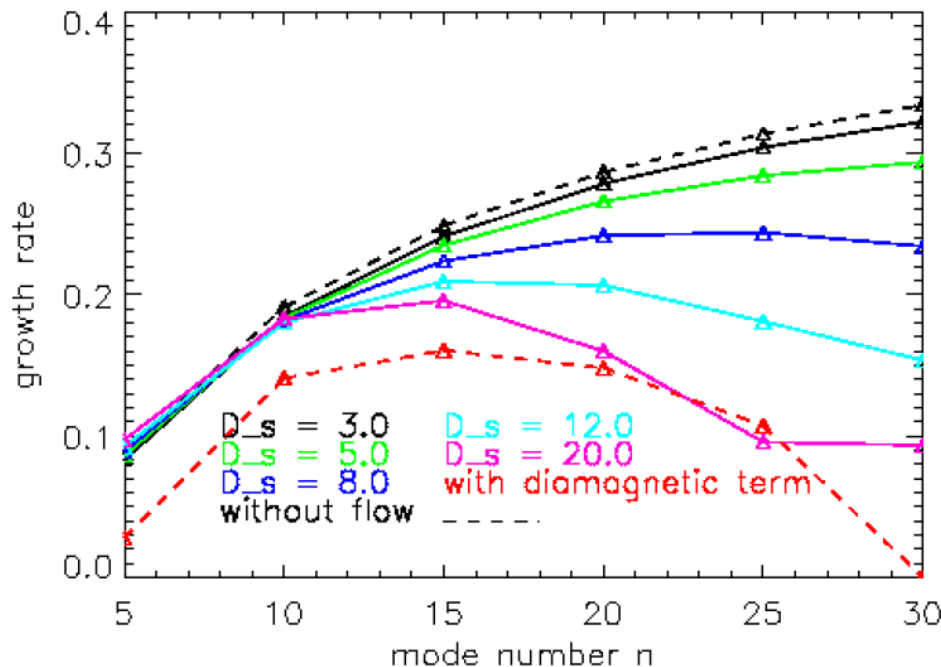
➤ **Diamagnetic drift:** inversely depends on density;

● **Net flow:** perpendicular component of toroidal rotation, modeled by a simple function via  $\Phi_{v0}$ , flexible;

● **Kelvin-Helmholtz term:** curl of net flow, can be switched off;

● **Total convection flow:** flow shear effects come from this total convection flow rather than the net flow.

# Equilibrium flow shear can be a double-edged sword on P-B modes



- The flow shear plays the same role as diamagnetic stabilization for ideal MHD case without diamagnetic term.
- Kelvin-Helmholtz drive mainly destabilize intermediate  $n$  modes:  $n=10\sim30$ .

# 5-field Peeling-Ballooning model

- In order to investigate the separate effects of density and temperature effect, we extend the 3-field simple P-B model into 5-field model by separating the total pressure into density electron and temperature

$$\frac{\partial n_i}{\partial t} + \mathbf{V}_E \cdot \nabla n_i = 0,$$

$$\bar{\omega} = n_{i0} \frac{m_i}{B_0} \left[ \nabla_{\perp}^2 \phi + \frac{1}{n_{i0}} \nabla_{\perp} \phi \cdot \nabla_{\perp} n_{i0} + \frac{1}{n_{i0} Z_i e} \nabla_{\perp}^2 p \right],$$

$$\frac{\partial T_j}{\partial t} + \mathbf{V}_E \cdot \nabla T_j = 0,$$

$$J_{\parallel} = J_{\parallel 0} - \frac{1}{\mu_0} \nabla_{\perp}^2 (B_0 \psi),$$

$$\frac{\partial}{\partial t} \bar{\omega} + \mathbf{V}_E \cdot \nabla \bar{\omega} = B_0^2 \mathbf{b} \cdot \nabla \frac{J_{\parallel}}{B_0} + 2 \mathbf{b} \times \kappa \cdot \nabla P,$$

$$\mathbf{V}_E = \frac{1}{B_0} (\mathbf{b}_0 \times \nabla_{\perp} \Phi),$$

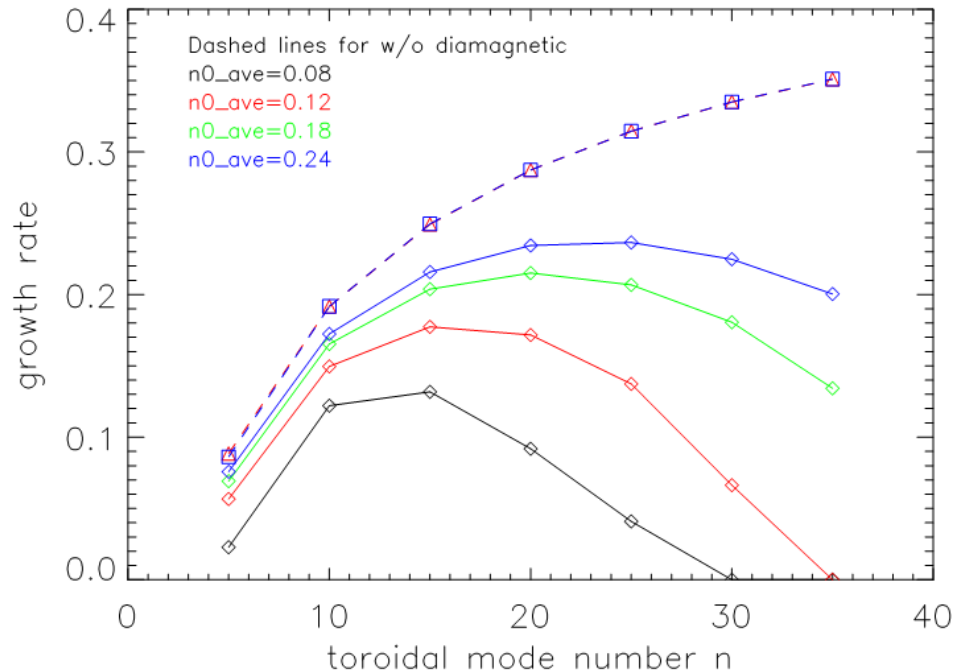
$$P = k_B n (T_i + T_e) = P_0 + p,$$

$$\frac{\partial \psi}{\partial t} = -\frac{1}{B_0} \mathbf{b} \cdot \nabla \Phi + \frac{\eta}{\mu_0} \nabla_{\perp}^2 \psi - \frac{\eta_H}{\mu_0} \nabla_{\perp}^4 \psi,$$

$$\Phi = \Phi_0 + \phi.$$

# The strong stabilizing density effect on P-B modes is due to ion diamagnetic drift

$n_0 = \text{constant in } x$ ,  $T_{e0}$  and  $T_{i0}$  vary in  $x$



- For ideal MHD,  $n_0$  does not affect the normalized linear growth rate.
- With diamagnetic effects,
  - low density results in more stable high-n modes.

$$n_{i0}(x) = \frac{(n0_{height} \times n_{ped})}{2} \left[ 1 - \tanh \left( \frac{x - x_{ped}}{\Delta x_{ped}} \right) \right] + n0_{ave} \times n_{ped},$$